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FORECASTING USING COMPLEX-VALUED AUTOREGRESSION WITH ERROR

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Abstract. This article discusses the possibility of predicting the values of a series using complex-valued autoregression with an error for short-term forecasting. The authors consider the basic concepts of the function of a complex-valued variable and the model of complex-valued autoregression, together with the results of applying first- and second-order models of complex-valued autoregression with the CARE(p) error to describe and predict the initial series. The results obtained are compared with the first- and second-order autoregression in real numbers. A complex-valued autoregression model with an error showed a more accurate result for short-term forecasting, unlike the autoregression model in real numbers. The authors also conclude that complex-valued autoregression with an error is subject to further investigation in order to find out the prospects of using its imaginary part.

Keywords: complex-valued autoregression with error, complex numbers, short-term forecasting, autoregressive model, standard deviation, function of complex-valued variable

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ПРОГНОЗИРОВАНИЕ С ИСПОЛЬЗОВАНИЕМ КОМПЛЕКСНОЗНАЧНОЙ АВТОРЕГРЕССИИ С ОШИБКОЙ

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Аннотация. В данной статье рассматривается возможность прогнозирования значений ряда с использованием комплекснозначной авторегрессии с ошибкой для краткосрочного прогнозирования. Рассматриваются основные понятия теории функции комплекснозначного переменного и модели комплекснозначной авторегрессии, приводятся результаты применения моделей первого и второго порядка комплекснозначной авторегрессии с ошибкой CARE(p) для описания и прогнозирования исходного ряда, сравниваются полученные результаты с результатами авторегрессии первого и второго порядков в действительных числах. В результате исследования, авторами был сделан вывод о возможности применения модели комплекснозначной авторегресии с ошибкой, так как она показала более точный результат для краткосрочного прогнозирования, в отличие от модели авторегрессии в действительных числах. Так же делается вывод, что комплекснозначная авторегрессия с ошибкой подлежит дальнейшему исследованию, чтобы выяснить возможность применения её мнимой части.

Ключевые слова: комплекснозначная авторегрессия с ошибкой, комплексные числа, краткосрочное прогнозирование, модель авторегрессии, среднеквадратичное отклонение, функция комплекснозначного переменного

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Introduction

Nowadays, humans often ask themselves the question, “What’s coming?”. We tend to find specific interest in the events that are coming even if it is impossible to look into the future. Many forecasters try to make their prognosis as close to reality as possible. Unfortunately, a forecast can never be one hundred percent correct, since numbers and formulas fail to describe our complex and constantly changing world. Still, everyone is trying to find a model that will be able to capture the trends of change.

As the volume of data in today's information society grows, forecasting plays an important role in various fields, from economics and finance to data science and marketing. Forecasting is an integral part of our lives. Every day we attempt to predict the outcome of various events and calculate our chances for all possible options. We even check out the weather forecast before going outside.

There are many different models that are capable of “predicting” the future to varied degrees. This research focuses on modelling stationary reversible processes that can be predicted using autoregressive models. Today, autoregressive models are the most frequently used ones in short-term economic forecasting (Svetunkov, 2021). For some reason, most forecasters use real numbers for their forecasts, considering complex numbers to be the prerogative of physicists and mathematicians (Doronin, 2023; Andrei, 2021). It does make sense, since complex numbers consist of two parts: real and imaginary. However, what is the imaginary part in the real world? Most forecasters probably stumble at this question and get back to the well familiar real numbers. For example, when modelling production processes, production functions of complex variables describe these processes in more detail and, in a number of cases, demonstrate greater accuracy than production functions of real variables (Svetunkov, 2019).

In this paper the authors briefly review the theory of complex-valued variables and examine the application of complex-valued autoregressive models such as CARE.

Materials and Methods

A complex number is a pair of numbers consisting of two parts – real and imaginary:

$$Z = x + iy$$

where,

x is the real part of a complex number;

iy is the imaginary part of a complex number;

x and y are real numbers;

i is the imaginary unit, which satisfies the equality: $i = \sqrt{-1}$

Complex numbers are quite an interesting tool that gives more possibilities, unlike real numbers. At the same time, it is possible to carry out all the same operations with real numbers as with complex numbers (Vasilyeva, 2019). Why then, is this tool not as popular among forecasters? It happens to be all about the imaginary part. Many do not understand its meaning in the real world, which is clear, because there is nothing imaginary in our world, only real. In fact, complex numbers are simply a tool for reflecting reality, like any procedure in mathematics, so almost any characteristic of a process can become imaginary.

A complex number consists of a pair of numbers. Thus, in order to reflect it on a plane, two numerical axes must be used. The real part of the number is plotted along the abscissa axis, and the imaginary part along the ordinate axis. Figure 1 shows a complex number in a Cartesian coordinate system.

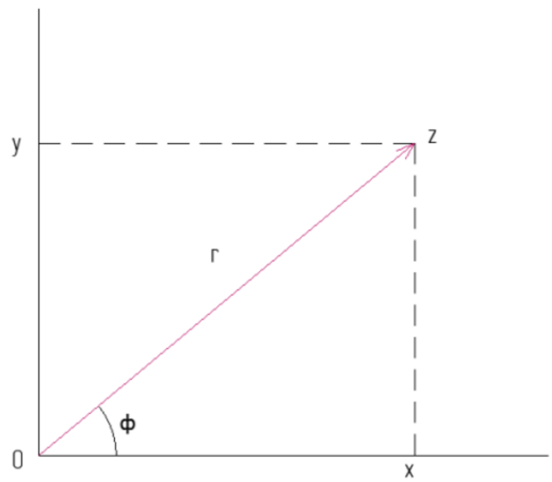

Fig. 1. Complex number in Cartesian coordinate system.

A complex number can be represented as a vector that starts at the origin and ends at point Z. Then any complex number can be represented in polar coordinates:

$$z = x + iy = r(\cos\varphi + i\sin\varphi) \quad (2)$$

where,

φ is the polar angle;

r is the polar radius (vector length), which is called the modulus of a complex number (Zvereva, 2021).

The modulus of a complex number is equal to:

$$r = \sqrt{x^2 + y^2} \quad (3)$$

The polar angle can also be easily found:

$$\varphi = arctg\frac{y}{x} + 2\pi k \quad (4)$$

where,

k is an integer. Sometimes the polar angle is called the argument of a complex number. In most cases, the condition k = 0 is accepted.

The fact that complex numbers can be considered simultaneously in both Cartesian and polar coordinates is their advantage over real numbers. Another important property of complex numbers, which will be useful to us for further research, is that two complex numbers are equal to each other if and only if their real and imaginary parts are equal to each other:

$$y_r + iy_i = F(y_r + iy_i) = f_r(x_r) + if_i(x_i) \quad (5)$$

We will consider the following form as a more compact:

$$\begin{cases} y_r = f_r(x_r) \\ y_i = f_i(x_i) \end{cases} \quad (6)$$

It is important to note that in this form of notation we have gotten rid of the imaginary unit, and for those who are not familiar with the theory of functions of a complex-valued variable, it will be easier to perceive this equality (the imaginary unit remains only in the indices to distinguish between variables). Modern researchers who use random variables in their work always believe that their parts (real and imaginary) are independent of each other (Tavares, 2006; 2007; Kennedy, 2008). Some scientists also consider the dependence of these parts, although they add the prefix "pseudo" – pseudo covariance or pseudo dispersion (Kammeyer, 2002; Picinbono, 2009). S.G. Svetunkov in his works (Svetunkov, 1999; 2019) demonstrated the conditions for the dependence of these two parts.

More information on the theory of functions of a complex-valued variable can be obtained from the manual "Complex numbers and functions of a complex variable" (Gamova, 2022; Peca, 2011). For further research in the framework of this paper, the presented information is more than sufficient.

Results and Discussion

Complex-valued autogression model

Let us examine models of complex-valued autoregressions for short-term forecasting (Svetunkov, 2021).

In general, the complex autoregressive model can go as follows:

$$y_{1t} + iy_{2t} = \sum_{\tau=1}^{p} F(y_{1(t-\tau)} + iy_{2(t-\tau)}) + (\varepsilon_{1t} + i\varepsilon_{2t}) \quad (7)$$

where,

y_{1t} and iy_{2t} are the real variables predicted at time t;

i is the imaginary unit, $i = \sqrt{-1}$;

F is some complex-valued function;

τ is the autoregressive lag;

p is the autoregressive order;

ε_{1t} and ε_{2t} are the approximation errors of the first and second variables at time t.

Depending on the type of function F, complex autoregressions can be linear and nonlinear. Nonlinear autoregressive models of real variables are not often encountered in either practical applications or theoretical studies. Therefore, we will use linear autoregressions and denote them as CAR(p). Thus, the considered complex-valued autoregressive models CAR(p) in general will be presented in the following form:

$$y_{1t} + iy_{2t} = (b_0 + ib_1) + \sum_{\tau=1}^{p} (a_{0\tau} + ia_{1\tau})(y_{1(t-\tau)} + iy_{2(t-\tau)}) + (\varepsilon_{1t} + i\varepsilon_{2t}) \quad (8)$$

where,

b_0 and b_1 are coefficients (free terms) reflecting the initial value of the complex series;

$a_{0\tau}$ and $a_{1\tau}$ are proportionality coefficients.

If we center the variables relative to the arithmetic mean, we can get rid of the free terms.

Then, the first-order complex autoregression CAR(1) can be represented as:

$$\hat{y}_{1t} + \hat{y}_{2t} = (a_{01} + ia_{11})(y_{1(t-1)} + iy_{2(t-1)}) \quad (9)$$

In this model, two variables are predicted. We transform this model to predict one variable, taking the second variable equal to the error ε_t, because it can serve as a characteristic of the process. Our model will take the following form:

$$\hat{y}_t + \hat{\varepsilon}_t = \sum_{\tau=1}^{p} (a_{0\tau} + ia_{1\tau})(y_{t-\tau} + i\varepsilon_{t-\tau}) \quad (10)$$

This model is usually refered to as a complex-valued autoregression with error and is denoted by CARE(p).

The idea of such a complex model was put forward first by I.S. Svetunkov back in 2011, when the complex-valued form of the exponential smoothing model was presented (Svetunkov, 2012; Racine, 2019). Research has shown that the complex-valued exponential smoothing model provides more accurate economic forecasts compared to exponential smoothing models of real variables (Svetunkov, 2015).

The CARE(p) model, as discussed earlier, can be represented as a system of two real variables:

$$\begin{cases} \hat{y}_t = \sum_{\tau=1}^{p} (a_{0\tau} y_{t-\tau}) - \sum_{\tau=1}^{p} (a_{1\tau} \varepsilon_{t-\tau}) \\ \hat{\varepsilon}_t = \sum_{\tau=1}^{p} (a_{0\tau} \varepsilon_{t-\tau}) + \sum_{\tau=1}^{p} (a_{1\tau} y_{t-\tau}) \end{cases} \quad (11)$$

Then for the real part of the complex-valued model we get:

$$\text{ReCARE: } \hat{y}_t = \sum_{\tau=1}^{p} (a_{0\tau} y_{t-\tau}) - \sum_{\tau=1}^{p} (a_{1\tau} \varepsilon_{t-\tau}) \quad (12)$$

And for the imaginary part of this model:

$$\text{ImCARE: } \hat{\varepsilon}_t = \sum_{\tau=1}^{p} (a_{0\tau} \varepsilon_{t-\tau}) + \sum_{\tau=1}^{p} (a_{1\tau} y_{t-\tau}) \quad (13)$$

Thus, in economic forecasting, in addition to the general CARE(p) model, two other independent models can be used: ReCARE(p) and ImCARE(p) (Svetunkov, 2020).

Using the CARE(p) model and comparing the CARE(1) and CARE(2) models

As mentioned earlier, the CARE(p) model can be used as a system of two equalities:

$$\text{ReCARE:}\ \hat{y}_t = \sum_{\tau=1}^{p}(a_{0\tau} y_{t-\tau}) - \sum_{\tau=1}^{p}(a_{1\tau}\varepsilon_{t-\tau}) \quad (14)$$

$$\text{ImCARE:}\ \hat{\varepsilon}_t = \sum_{\tau=1}^{p}(a_{0\tau}\varepsilon_{t-\tau}) + \sum_{\tau=1}^{p}(a_{1\tau} y_{t-\tau}) \quad (15)$$

These models describe a series with some error, and the problem of estimating the coefficients $a_{0\tau}$ and $a_{1\tau}$ is reduced to minimizing the sum of the squares of this error. For models (14) and (15), these errors are respectively equal to:

$$\text{Re:}\ \varepsilon = y_t - \hat{y}_t \quad (16)$$

$$\text{Im:}\ \mu = \varepsilon_t - \hat{\varepsilon}_t \quad (17)$$

Moreover, if we minimize the sum of squares (16), then the errors (17) will be very large, and vice versa.

For the study, series No. 2810 was taken from the International Institute of Forecasters database. For this series, forecast values ($\hat{y}_t$) and forecast deviations ($\hat{\varepsilon}_t$) were calculated using the CARE(1) and CARE(2) models.

The results obtained are presented in Figure 2.

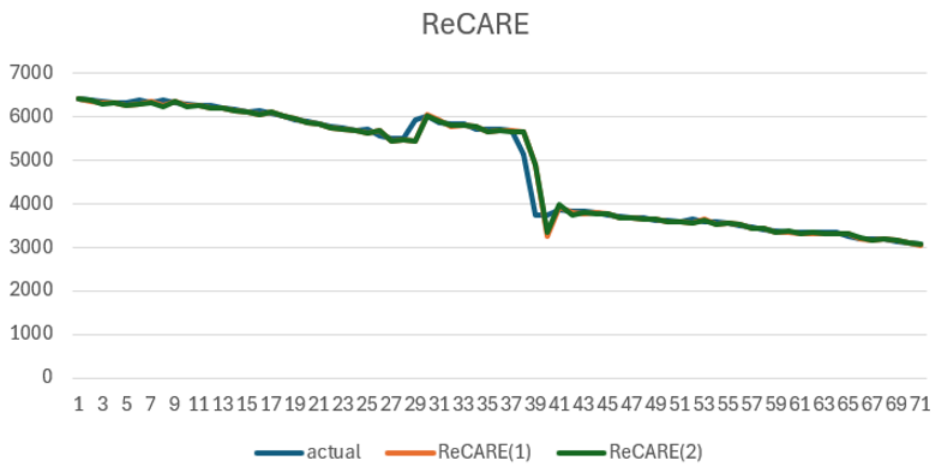

Fig. 2. Initial series y_t and calculated values of the model ReCARE(p).

The model, as can be seen from the figure, described the series well and was able to capture the trends of change. There is no particular difference between the models ReCARE(1) and ReCARE (2).

The results obtained for the model ImCARE(p) are presented in Figure 3.

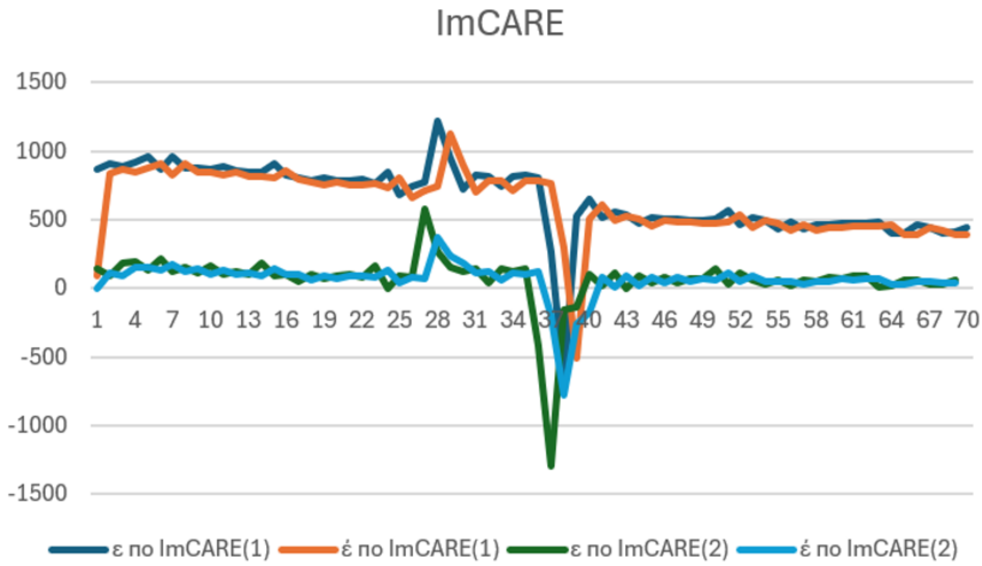

Fig. 3. Deviations ε_t and calculated values of model deviations ImCARE(p).

Models ImCARE(1) and ImCARE(2) also coped well with the task. They adequately described the deviations of their own models. However, these models described errors ε_t, with absolutely different coefficients $a_{0\tau}$ and $a_{1\tau}$ from models ReCARE(1) and ReCARE(2). Here, the question does arise: "How did the calculated values y_t deal with the coefficients $a_{0\tau}$ and $a_{1\tau}$ of models ReCARE(1) and ReCARE(2)?" In order to respond, it is necessary to minimize the sum of the squares of the approximation error μ (17) in models ReCARE(1) and ReCARE(2). Figure 4 depicts the obtained results.

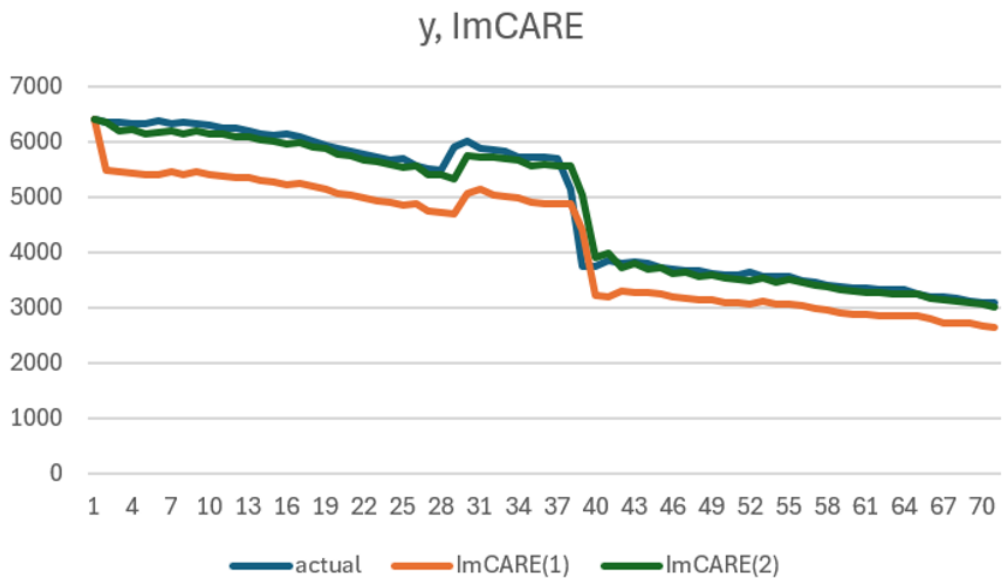

Fig. 4. Initial series y_t and calculated values for the model coefficients ImCARE(1) and ImCARE(2).

These models also capture the trend of change, but it is as if the model with ImCARE(1) coefficients is shifted along the ordinate axis below the actual values by some fixed value. The model with ImCARE(2) coefficients described the original values y_t way better.

The only remaining thing to do is to find out what calculated values of deviations ε_t will be obtained when using the coefficients of the ReCARE(1) and ReCARE(2) models. In order to do so, the ImCARE(1) and ImCARE(2) models will be used, but the sums of the squares of the error ε (16) will be minimized. Figure 5 demonstrates the results.

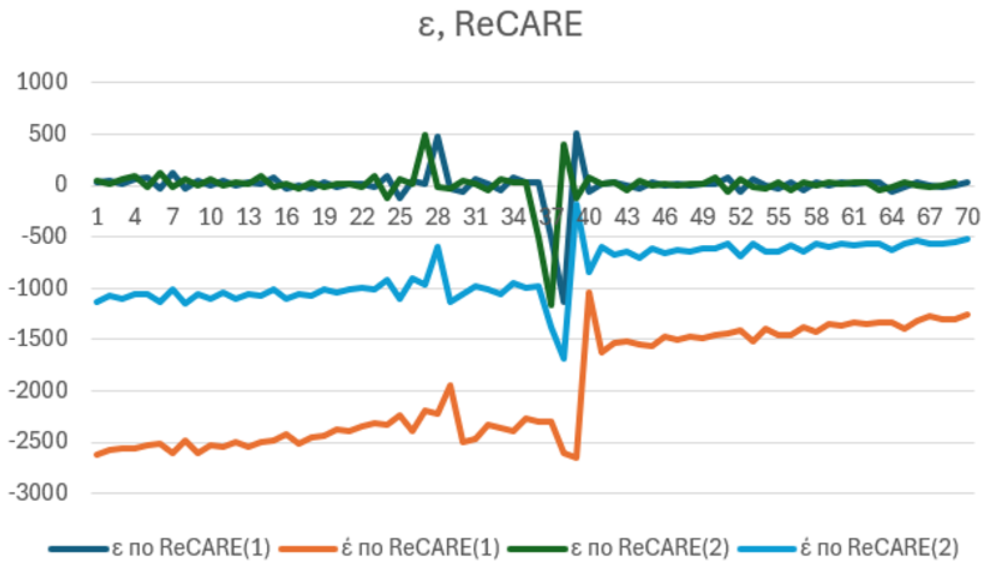

Fig. 5. Deviations ε_t and calculated values of deviations for the coefficients of models ReCARE(1) and ReCARE(2).

It is no surprise that the deviations of the two models are very close, since their calculated values are also close (Fig. 2). What is more interesting is that the calculated values of the deviations ε_t capture the trend of change, but again there is a shift along the ordinate axis lower by some value.

It is difficult to describe the scope of application of the ImCARE (p) model, since it does not predict the values of y_t, but the characteristics it calculates can probably serve as an additional characteristic of the process under study, which remains unclear.

For further research, we took the same series №2810 from the International Institute of Forecasters database. This series consists of 71 observations. This time we will split it into two parts – training and testing. The first 67 observations will be training, and the coefficients of the ReCARE(1) and ReCARE(2) models will be calculated on them. The last 4 observations will serve as testing for these models. The later will be compared by calculating the relative error. Table 1 summarizes the results.

Table 1. Comparison of ReCARE(1) and ReCARE(2) models

Observation No.	y	1 order		2 order	
		$\hat{y}_t$ Re	Error, %	$\hat{y}_t$ Re	Error, %
68	3181.4	3179.938	0.045955939	3202.492	0.66299386
69	3129.1	3135.165	0.193818401	3146.096	0.543160288
70	3086.6	3107.248	0.668957055	3116.743	0.976576857
71	3081.5	3077.714	0.122854666	3084.117	0.084934346

The table shows that both models coped with the task and were able to predict the next values with an accuracy of over 99%. The ReCARE(1) model did a better job, although the ReCARE(2) model also performed well.

Now, using the same principle, we will compare the ImCARE(1) and ImCARE(2) models, only the calculated values will be compared with the deviation ε, each model will have its own deviations, since the calculated values y_t differ. The results are given in Table 2.

Table 2. Comparison of ImCARE(1) and ImCARE(2) models

Observation No.	1 order			2 order		
	ε Im	$\hat{\varepsilon}$ Im	Error, %	ε Im	$\hat{\varepsilon}$ Im	Error, %
68	264.7697	278.7766	5.290202952	45.54745	33.06735	27.40020701
69	3296.899	3294.786	0.059053573	216.462	217.8235	0.628957094
70	3359.584	3358.786	0.023741284	1703.596	1755.585	3.051706806
71	3457.8	3417.41	1.168089727	3466.573	3411.657	1.584169878

The ImCARE(1) model showed a more accurate result. The ImCARE(2) model also showed a good result, except for the 68th observation. It should also be noted that both models showed absolutely different results on the same observations, with the difference reaching the order of thousands. Moreover, ε increases in both cases from 68 to 71 observations. The deviations of ε itself are smaller when using the second-order model.

In this regard, the behavior y_t of the coefficients of the ImCARE(1) and ImCARE(2) models became interesting. The comparison was carried out according to the same principle. Table 3 presents the obtained results.

Table 3. Comparison y_t of the coefficients of the ImCARE(1) and ImCARE(2) models

Observation No.	y	1 order		2 order	
		$\hat{y}_t$ Im	Error, %	$\hat{y}_t$ Im	Error, %
68	3181.4	2916.63	8.322427811	3135.853	1.431679331
69	3129.1	-167.799	105.3625424	2912.638	6.917709369
70	3086.6	-272.984	108.8441611	1383.004	55.19328402
71	3081.5	-376.3	112.2115983	-385.073	112.4962837

Surprisingly enough, 68 and 69 observations of the 2nd order model can be considered quite good. For a more detailed analysis, graphs are presented below (Fig. 6).

Fig. 6. Initial series y_t and calculated forecast values for model coefficients ImCARE(1) and ImCARE(2).

As can be seen from the figure, these models described the training set quite well up to the 68th observation (although the ReCARE(p) model did better). Starting from the 69th observation, the model with the ImCARE(1) coefficients went into disarray. The model with the ImCARE(2) did so starting from the 71st observation, although it started showing poor results already at the 70th observation.

Comparing the deviations and their error using the coefficients of the ReCARE(1) and ReCARE(2) models does not make any sense, since when using these models they did a poor job in describing the original series (Fig. 5).

Comparison of AR(p) and CARE(p) models

The general form of the autoregressive model AR(p) is:

$$\hat{y}_t = \sum_{\tau=1}^{p} a_\tau y_{t-\tau} \tag{18}$$

With its help, stationary processes are modeled, each current value of y_t which is determined by previously accumulated values y_{t-1}, y_{t-2}, etc.

In order to construct this model, and namely to determine the values of the coefficients a_τ, as for the ReCARE model, it is necessary to minimize the sum of the squares of the deviations ε:

$$\varepsilon = y_t - \hat{y}_t \tag{19}$$

As can be seen from the model, it is not oriented towards taking into account the approximation errors ε.

To begin with, let's construct a first-order model, which will take the form:

$$\hat{y}_t = a_1 y_{t-1} \tag{20}$$

For the analysis we will use the same series #2810 from the International Institute of Forecasters database.

Figure 7 presents the obtained results.

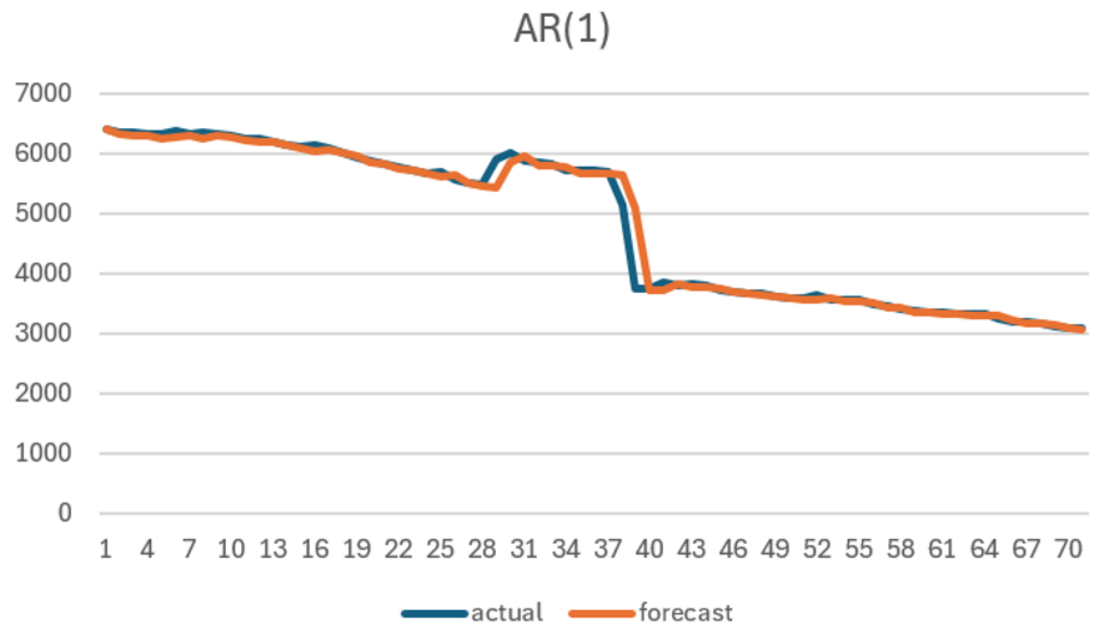

Fig. 7. Initial series y_t and calculated values of the model AR(1).

Overall, the model did a good job. It described the series perfectly well and captured the trend of changes. However, the ReCARE model will be able to compete with it, since Figure 7 and Figure 1 are quite similar.

Next, we will build a second-order autoregressive model, it will take the form:

$$\hat{y}_t = a_1 y_{t-1} + a_2 y_{t-2} \tag{21}$$

Figure 8 presents the obtained results.

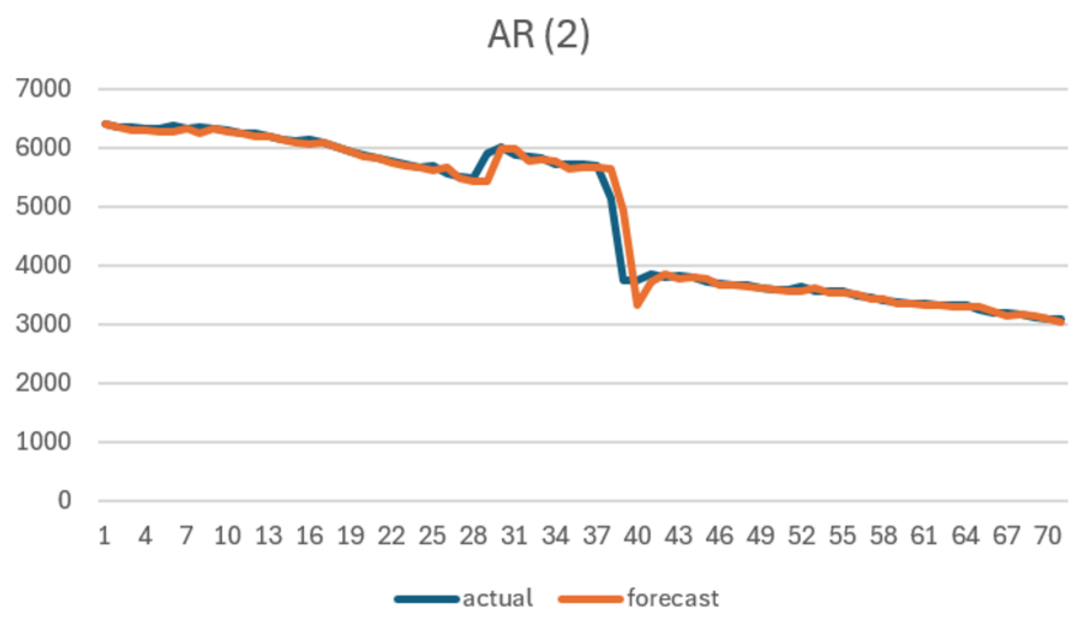

Fig. 8. Initial series y_t and calculated values of the model AR(2).

No significant changes are observed. The model performed well, as did the models in Figures 1 and 7.

The ReCARE model performed no worse or better than the well-known AR model, meaning that the ReCARE model is competitive. Now, let's compare the ReCARE(p) and AR(p) models in numerical values.

In order to compare two different models, we need a more precise criterion than just a single-strength error. Deviations from the test set are not always the best indicator for choosing a model, since only the last values are compared, not the entire series.

In order to compare the results, three most commonly used approximation accuracy characteristics were calculated:

1. Root mean square deviation of the approximation error (RMSD);
2. Akaike information criterion (AIC);
3. Bayesian information criterion (BIC).

The standard deviation of the error shows how large or, on the contrary, insignificant the errors of approximation of the entire series were. The smaller RMSD – the better. The last two criteria show the "clutter" of the model; by comparison, they can be used to determine a simpler model that retains the accuracy results.

First, as with the ReCARE(1) and ReCARE(2) models, we will split the data into training and testing data, and then check how the AR(1) and AR(2) models performed and compare them with other models based on the specified criteria. Table 4 summarizes the obtained data.

Table 4. Comparison of approximation accuracies of the ReCARE(p) and AR(p) models

AR						
Observation	**RMS (1st order)**	AIC (1)	BIC (1)	**RMS (2nd order)**	AIC (2)	BIC (2)
68	**192.1437**	10.54589868	10.5785385	**185.3886**	10.50373156	10.5690112
69	**304.6962**	11.46761622	11.49999457	**305.2839**	11.50045567	11.56521238
70	**371.4359**	11.86332406	11.89544542	**373.6194**	11.90361787	11.96786059
71	**420.8575**	12.11275749	12.14462622	**423.5752**	12.15380011	12.21753758
ReCARE						
Observation	**RMS (1st order)**	AIC (1)	BIC (1)	**RMS (2nd order)**	AIC (2)	BIC (2)
68	**180.18735**	10.4468178	10.51209748	**172.05523**	10.4132781	10.54383738
69	**304.6941**	11.4965877	11.56134439	**301.72071**	11.5349456	11.66445901
70	**375.75992**	11.9150437	11.97928646	**372.20506**	11.9531756	12.08166102
71	**420.75671**	12.1404476	12.20418503	**422.58824**	12.2054726	12.33294752

All models showed the best result on the first observation, which is understandable, because the problem is short-term forecasting. In fact, all models coped with forecasting specific observations with approximately equal accuracy. The ReCARE(2) model showed the best results in forecasting the 68th (although the ReCARE(1) model also coped better than the models of ordinary autoregression). This indicates the accuracy of the ReCARE model.

Conclusion

In this research, the theory of complex-valued variable functions was considered, and the model of complex-valued autoregression of the CARE(p) was studied. Autoregression of this type consists of two parts: ReCARE(p) and ImCARE(p).

The ReCARE(p) model is excellent at learning from the given data, picking up on trends, and being able to “predict” the data with excellent accuracy.

The ImCARE(p) model is well-oriented to predict approximation errors, but with completely different coefficients from the ReCARE(p) model. The model is well trained to predict the values themselves but is only capable of producing good results for one step, while the ReCARE(p) model with its coefficients produces a result much more accurately. This model needs to be studied in more detail to recognize its practical application.

When comparing the ReCARE(p) model with the standard autoregressive (AR) model, the ReCARE(p) model showed a more accurate result when forecasting for one step; the standard deviation was 180.187 (first order) and 172.055 (second order) against 192.144 and 185.389, respectively. This indicates the advantage of the complex-valued autoregressive model. These two models are short-term forecasting models, so when forecasting for more steps than one, they showed slightly worse results. Nevertheless, the accuracy of these results is quite high; the models coped with the task at about the same level.

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